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The transport equation theory applied to the prediction of the spatial distribution of an electron beam in a target

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Abstract. The aim of this paper is to make a contribution to the understanding of the phenomenon of the electron distribution under the circumstances of a beam-induced charge build-up in a target. Such phenomena are encountered in electron-beam-assisted microanalysis or in Auger electron analysis. The theoretical problem of the motion of a beam of charged particles in a solid is resolved by interpreting the transport equation in terms of an integro-differential equation. A treatment of the Boltzmann equation is developed for the energy dependence in order to predict the electron penetration in targets bombarded under the simultaneous influence of scattered and decelerated incident particles. The exact values of the specific moments of the electron distribution in a semi-infinite medium are calculated. We suggest a procedure applied to the calculation of an analytic expression characterizing the asymptotic trend of the electron spatial distribution at very small residual energies. The perturbing effects of insulator irradiation are treated and charge deposition profiles near the surface are calculated for mixed alkali silica glasses. Results are compared with previous data; agreement is in most cases fairly good.

1. Introduction

In the last few years several techniques using electron beams have been increasingly developed for investigations concerning condensed matter or the solid state. They are based on quantitative or qualitative information from secondary radiation excited by an electron bombardment. They concern, for example, the unknown elemental concentration of a polyatomic sample. Correction factors are necessary in electron probe microanalysis for extracting the true information concerning the specimen. They consist of three terms, which account for atomic number, absorption and fluorescence effects. As the analysed x-rays are emitted from electrons travelling in the solid, the electron distribution with depth will have to be taken into account in the necessary correction set. A number of difficulties arise, principally in the case of the analysis of the specimen, which is modified under electron irradiation. Because of unresolved problems and untested hypotheses the correction procedures in the presence of the electric field arising from the charge build-up require rigorous investigations into three features of the problem:

(i) the first, definitely predominant, concerns the penetration of electrons in insulating targets, which depends strongly on the charge produced at the target surface;

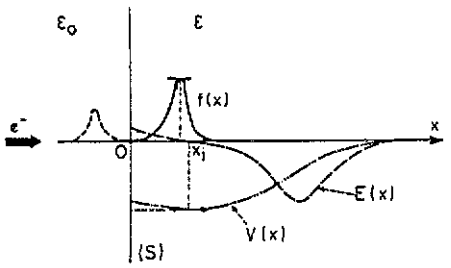

Figure 1. Schematic representation of electron beam penetration into a target; $f(x)$ is the depth distribution of the electrons in the target. The depth distribution of the electric potential $V(x)$ and the electric field $E(x)$ in the specimen are created by the negative charges in the insulator and their images located at the front of the target.

(ii) the second refers to the ionic mobility of some cations—the $^{23}Na(p, \alpha)^{20}Ne$ nuclear reaction has been used by Battaglin *et al* [1] to investigate the depth profile of sodium near the surface of soda lime silicate glasses; they have shown after electron irradiation an accumulation of sodium near the surface at a depth approaching that of the electron penetration;

(iii) the third relates to the stability of bonds in ionic solids—Knotek and Feibelman [2] have provided criteria for compounds in which the cation and anion have Pauling electronegativity differences greater than 1.7; such solids are, for example: MoO_3–SiO_2–Al_2O_3–TiO_2–V_2O_5–, etc.

Many authors [3, 4] have studied in detail the general problem of multiple scattering. Specific analytical solutions have been given in an approximate way for several cases such as the penetration of electrons [5], neutrons [6] or α-particles [7] in solids, and radiative transfer in stars [8]. Various theories have been developed for particular geometrical configuration of the target: infinite [9], semi-infinite [10] or slab [11]. Isotropic [12] or anisotropic [13] scattering, plane or point sources [14] have also been discussed. More recently Biagy [15] has written a computer program that solves the Boltzmann equation for particles propagating in gases in the presence of both an electric and a magnetic field. At least three different approaches to the problem of multiple scattering of electrons have been published in the literature:

(a) a simple combination of the paths that are followed by incident electrons upon entering the solid target with convenient statistical weights [16];

(b) a simulation of many individual particle trajectories by the Monte Carlo method [17];

(c) a resolution of the fundamental Boltzmann transport equation [18], where the attempt to follow the motion of individual electrons is abandoned, being replaced by the treatment of the behaviour of an electron flow.

The main objective of our investigation is to determine precisely the manner in which the transport equation can be applied to the electron–specimen interaction in the case of conducting or insulating materials, which are bombarded by particles with an incident energy in the range 3–30 keV as illustrated by figure 1. This situation is of peculiar interest because monokinetic electron beams in this energy range are used in many commercial spectrometers such as electron probe microanalysers. An important phenomenon must be considered in all the techniques using charged-particle beams applied to the analysis of a great variety of materials such as insulators: in this case the electric field created by the trapped-charge distribution modifies the electron penetration profile strongly. In order to avoid the spurious effects of the charging of sample on the analysis of the results a general model for calculating the potential distribution has been

developed. It is based on the dynamic interaction between the primary charge in the specimen while the secondary electrons yield positive charges near the target surface. Because of recombination effects of positive and negative charge densities, the variation in trapped charges is not a linear function of time. By using Auger lines, it is possible to measure the surface potential of the target: Vigouroux *et al* [19] have shown that the potential in the bombarded area of pure amorphous silica depends on the primary energy and can reach several keV. The charge build-up of the sample has at least two main effects on the electron propagation, as demonstrated by Monte Carlo calculations [20, 21]; it decreases the energy of incident particles and modifies the electron trajectories in such a way that they tend to be deflected in a direction opposite to the electric field. The consequence is a decrease of the penetration depth of the incident particles. A simple approach describes an analytical method based on the determination of the solution of the diffusion equation. Backscattered and secondary electrons are taken into account for the discussion of the phenomenon of electromigration.

2. The transport equation

We consider the following problem: an axially symmetrical beam of electrons is normally incident on the plane surface of a semi-infinite amorphous target. As the electrons move away from the source, they lose energy and change their direction. We shall be concerned with the steady state distribution of the scattered particles. As in any problem of multiple scattering two steps have to be considered:

(i) the basic event of single scattering, which is characterized by the angular scattering law and the energy loss law;

(ii) the solution of the statistical problem of calculating the distribution of electrons after a large number of elastic or inelastic collisions. We follow the usual simplifying procedure of ignoring the straggling process in the energy loss of electrons and we treat it separately from the angular diffusion. The continuous slowing down approximation makes it possible to use the path length as state variable instead of energy.

In this paper the Boltzmann transport equation has been used to calculate the electron distribution after elastic and inelastic scattering processes in conducting or insulating targets. This continuity equation is based on the general theory of conservation of particle number [22]: the evolution with time of the electron distribution $f(\boldsymbol{x}, \boldsymbol{v}, s)$ is represented by a vector in a six-dimensional space; one component arising from convection is given by $\boldsymbol{v}f(\boldsymbol{x}, \boldsymbol{v}, s)$ and one component is determined by the applied forces $e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})\, f(\boldsymbol{x}, \boldsymbol{v}, s)$, including the effects of an electric field $\boldsymbol{E}$ and a magnetic field $\boldsymbol{B}$. Electron charge is given by e, $\boldsymbol{x}$ and $\boldsymbol{v}$ are the position and the velocity vectors, respectively, and s is the path length traversed by the electrons (s is a scalar function of time, $s(t)$). In what follows we shall only treat the motion of electrons in a medium characterized by the presence of an electric field having a longitudinal gradient, in the absence of any magnetic field. The effect of the collisions is to transfer electrons from one element in velocity space to another. If we suppose continuous forces, we can write:

$$\frac{\partial f}{\partial t} + \operatorname{div}_x(\boldsymbol{v}f) + \operatorname{div}_v(e\boldsymbol{E}f) = N \int [f(\boldsymbol{x}, \boldsymbol{v}', s) - f(\boldsymbol{x}, \boldsymbol{v}, s)]\, \sigma(|\boldsymbol{v}' - \boldsymbol{v}|)\, \mathrm{d}\boldsymbol{v}. \tag{1}$$

Here N is the density of scattering centres and $\sigma(\mu)$ is the well known scattering cross section per unit solid angle. For simplicity in describing the angular scattering we use

the Rutherford differential cross section with the screening parameter given by Mott and Massey [23]:

$$\sigma(\mu) = (Z^2e^4/4W_2)(1/(1 + \mu + 2\eta)^2) \tag{2}$$

where Z is the atomic number, W is the electron energy and η accounts for the screening of the nucleus by orbital electrons:

$$\eta = me^4Z^{2/3}/8W\hbar^2 \tag{3}$$

where m is the rest mass of the electrons and h is the Planck constant, $h = \hbar/2\pi$. For a beam of electrons, considered as uniform in the y- and z-directions, equation (1) simplifies to

$$\frac{\partial g}{\partial s} + \mu\frac{\partial g}{\partial x} + \mu eE\frac{\partial g}{\partial W} + \frac{eE}{2W}(1 - \mu^2)\frac{\partial g}{\partial \mu} = N\int [g(\mu') - g(\mu)]\sigma(\mu)\,\mathrm{d}\mu. \tag{4}$$

The angular coordinate μ is the cosine of the angle defined by the trajectory of an electron and by its initial velocity, and E is the electric field, which is variable along the x-axis. μ is the cosine of the scattering angle.

If the approximation of small-angle electron scattering is introduced at this stage one can expand the solution of the integro-differential transport equation in Bessel functions [24] whereas, in the general case of electron propagation in an infinite medium, an expansion in Legendre polynomials [25] is required. We have used an approximate expression for determining an analytical expression of the electron distribution for multiple scattering through infinite and semi-infinite targets.

3. Electron distribution in an infinite conducting target

We first examine the distribution as a function of depth for electrons deposited in an uncharged target; the primary beam is defocused. The transport of the charges from a source with a broad area in the presence of collisions can be described by the one-dimensional Boltzmann equation with solutions expanded in Legendre polynomials. Bethe *et al* [26] have shown that the electron density

$$F(x, s) = \int f(x, v, s)\,\mathrm{d}v\,\mathrm{d}y\,\mathrm{d}z$$

of the electrons at the depth x can be considered as a solution of the diffusion equation (5) in the lowest order approximation of the Boltzmann equation. This equation is obtained as the limit of the transport equation when the electron distribution is sufficiently broad. The diffusion approximation is valid provided that the primary beam becomes sufficiently scattered and that the stopping power is sufficiently weak, this assumption is satisfied if the trajectory has a length greater than about two fifths of the electron transport mean free path. Kanaya *et al* [27] have reported that, in accordance with the model of Bethe *et al*, the primary beam spreads with a Gaussian behaviour as the penetration increases. Following Bethe it is useful to introduce the current density

$J(x, s) = \int f(x, v, s)v \, dv \, dy \, dz$. The Boltzmann equation is then integrated over all three directions, thus:

$$\partial F/\partial s = -\text{div}\, J. \tag{5}$$

In the same manner the integration over all directions after multiplying the Boltzmann equation by v gives

$$\partial F/\partial s = -\tfrac{1}{3}\,\text{grad}\, F + J/\lambda \tag{6}$$

where λ is the usual electron transport mean free path which is defined as

$$\frac{1}{\lambda} = 2\pi N \int \sigma(\mu)(1-\mu)\, d\mu = \frac{\pi N e^4 Z}{2W^2} \ln \frac{2ma_h(2W/m)^{1/2}}{hZ^{1/3}}. \tag{7}$$

It is convenient to treat equation (7) by choosing s as a state variable instead of the energy W; we use the stopping power formula derived from the general theory of Bethe:

$$dW/ds = (2\pi N e^4 Z/W) \ln(2W/J). \tag{8}$$

We consider then that the propagation is assimilated to a diffusion process if F and J do not change considerably over one mean free path λ. Consequently we can neglect $\partial J/\partial s$ and combining equations (5) and (6) we obtain the diffusion equation:

$$\partial F/\partial s = \tfrac{1}{3}\lambda \Delta F. \tag{9}$$

We have introduced so far the Bohr radius a_h, which is the radius of the smallest electron orbital in the hydrogen atom, $a_h = \hbar^2/me^2 = 0.052917$ nm. Considering the results of Meister [28] the arguments of the logarithm in the expressions (7) and (8) change very little with energy between 3 and 30 keV for elements with atomic number $Z < 80$, we use a linear dependence of the mean free path with the path length. Substitution of the path length calculated from equation (7) into equation (9) yields the following formula for the electron transport mean free path:

$$\lambda = (s_0 - s)/c \tag{10}$$

where s_0 is the electron range in the target. It follows from equation (9) that the distribution function $F_i(x, s)$ of the electrons deposited in an infinite target is given by a Gaussian distribution if the residual energy of the electrons tends to zero. This is a fair approximation to the steady state electron distribution for electron implantation, as reported by Kanaya *et al* [27]:

$$F_i(x, s) = N_x(\langle x \rangle, \sigma_x). \tag{11}$$

$N_x(\langle x \rangle, \sigma_x)$ denotes here the Gaussian distribution where $\langle x \rangle$ is the average penetration of the electrons and σ_x its variance, $\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$. Below we note $\langle x \rangle$, the average value of a variable x, the integral:

$$\langle x \rangle = \int x f(x, v, s)\, dx\, dv.$$

We are in fact interested in the asymptotic trend of the electron spatial distribution. In order to calculate $\langle x \rangle$ and $\langle x^2 \rangle$, that is the first two moments of the spatial distribution in the infinite medium, the transport equation (4) is transformed into a double system (12) and (14) of linked differential equations, after a multiplication by μ and by x for the calculation of $\langle x \rangle$ and a multiplication by μ^2, by μx and by x^2 for the calculation of $\langle x^2 \rangle$, followed by an integration over all space.

3.1. Mean value of the electron penetration

$$\mathrm{d}\langle\mu\rangle/\mathrm{d}s + \langle\mu\rangle/\lambda = 0 \qquad \mathrm{d}\langle x\rangle/\mathrm{d}s = \langle\mu\rangle. \tag{12}$$

Resolving these equations, we obtain the following values for the mean penetration:

$$\langle\mu\rangle = \exp\left(-\int_0^s \frac{\mathrm{d}s}{\lambda(s)}\right) \qquad \langle x\rangle = \int_0^s \langle\mu\rangle\,\mathrm{d}s. \tag{13}$$

3.2. The second moment of the electron distribution

$$\mathrm{d}\langle\mu^2\rangle/\mathrm{d}s - 3\langle\mu\rangle/2\nu = 1/2\nu \qquad \mathrm{d}\langle x\mu\rangle/\mathrm{d}s - \langle x\mu\rangle/\lambda = \langle\mu^2\rangle \qquad \mathrm{d}\langle x^2\rangle/\mathrm{d}s = 2\langle x\mu\rangle \tag{14}$$

with

$$\frac{1}{\nu} = 2\pi N \int \sigma(\mu)(1-\mu^2)\,\mathrm{d}\mu. \tag{15}$$

Similarly we obtain for $\langle x^2\rangle$:

$$\langle x^2\rangle = 2\int_0^s \langle\mu\rangle \int_0^s \frac{\langle\mu^2\rangle}{\langle\mu\rangle}\mathrm{d}s'\,\mathrm{d}s. \tag{16}$$

Substitution of the mean free path calculated from equation (10) into equations (13) and (16) yields the following formula for the first moments of the distribution of electrons in an infinite medium:

$$\langle x\rangle = [1/(c+1)][s_0 - s_0(1 - s/s_0)^{c+1}] \tag{17}$$

$$\langle x^2\rangle = \frac{2}{3(c-1)}\left(s(s_0 - \tfrac{1}{2}s) - \frac{1}{c+1}s_0^2[1-(1-s/s_0)^{c+1}]\right). \tag{18}$$

4. Distribution of electrons in a semi-infinite conducting target

In order to link the study of multiple scattering in infinite media to the relevant real investigations we consider the concrete problem of electron transport in the presence of boundary conditions. Carslaw and Jaeger [29] have treated the diffusion equation (5) in various geometrical and initial conditions with considerable detail. Our treatment of the multiple-scattering problem is limited to the consideration of a semi-infinite medium with the boundary conditions described by Bethe and Jacob [30]. Carslaw and Jaeger suppose the medium to be continuous on the negative side of the surface. By using the process of images of the sources, they obtain boundary conditions for the semi-infinite diffusion problem whose solution is deduced from that of the infinite medium by the superposition theorem. The boundary condition on the electron density at the target surface is given by stating that the electron density at the free surface is proportional to the electron current flowing out of the target. Bethe and Jacob have shown that, if the electron distribution is sufficiently smooth, the boundary condition is given by equating the electron distribution at the extrapolated end-point to zero, that is $x = X_1 = -0.71\lambda$, $F_{si}(-0.71\lambda, s) = 0$. The source of electrons in an infinite medium may be considered at the mean value of the electron penetration $\langle x\rangle$. For the semi-infinite

medium, we put an image source of opposite sign at a point of abscissae $x_2 = 2x_1 - \langle x \rangle$, thus at the extrapolated end-point the electron distribution is equal to zero. We can then conclude that the distribution $F_{si}(x, s)$ of the electrons deposited in a semi-infinite target is given by the solution of equation (9):

$$F_{si}(x, s) = N_x(\langle x \rangle, \sigma_x) - N_x(x_2, \sigma_x). \tag{19}$$

Now, we investigate two important cases for electron spectroscopy analysis: the electric potential of the sample surface can have any value during electron-irradiation or is set at ground potential.

5. Electron distribution in a charged target without a conducting coating

Within the framework of Auger electron spectroscopy (AES) in this section, we are interested in the interaction of an electron beam that induces both surface and bulk charging effects on an insulator target. Since this technique involves detecting the Auger electrons it is sensitive to the first few atomic layers of the material and any change in analysis parameters influence strongly the AES data of insulators. Both Auger peak heights and energies are affected by the electron bombardment. During irradiation primary electrons penetrate inside the specimen to a depth of several microns while secondary electrons are emitted from the surface layer. The target surface then becomes positively charged under the usual operating voltage in electron probe microanalysis. The consequence of the interactions with the bulk is the creation of electron–hole pairs in the irradiated volume by the ionization of bound electrons or by excitation of conduction electrons followed by their trapping on localized levels. According to elementary theory, the number of electron–hole pairs released in the material is proportional to the energy loss of the incident electrons. The electrons ejected after inelastic collisions also interact with the atoms of the target. After a more or less long length of time a recombination process arises between the mobile primary electrons and the fixed positive charges created near the surface region. We now consider the mathematical analysis of this problem and we shall neglect for purposes of simplification some contributions of the interactions. Let us consider a semi-infinite insulator, with a given dielectric constant ε which is charged by an accumulation of electrons of the primary beam, the target being surrounded by a vacuum. The axial component of the electric field $E(x)$ induced in the specimen is calculated by using the Maxwell equation [31]. The procedure followed to determine the electric potential $V(x)$ consists in evaluating the contribution to $V(x)$ of the uniform density ρ of the primary electrons in the specimen and the density of their images symmetrically located in the vacuum characterized by the dielectric constant ε_0. It is evident that the assumption of a uniform charge density is not valid for an accurate determination of the potential near the target surface. Nevertheless it is justified by the fact that in our case the charge distribution calculated by using this hypothesis is in good agreement with the Monte Carlo results. The electric field is then

$$E(x) = \frac{\rho}{\varepsilon}\left(x - \frac{\varepsilon_0}{\varepsilon + \varepsilon_0} s_0\right) = \frac{\rho}{\varepsilon} y \qquad \text{with} \qquad y = x - \frac{\varepsilon_0}{\varepsilon + \varepsilon_0} s_0. \tag{20}$$

We deduce from this expression, (20), that near the surface a small decelerating field exists whose direction is opposite to that of the electron propagation. The variation in the electron penetration is therefore mainly the result of the potential distribution in the

region in front of the sample. These calculations are corroborated by the Monte Carlo simulation of Kotera and Suga [32] of the potential distribution in a charged polymethylmethacrylate wafer. The existence of an inversion point for the axial field with a small deceleration field and a high negative potential near the target surface is confirmed, if this surface is not set at ground potential. Returning to the original transport equation (4) and making the diffusion approximation we can write:

$$\partial^2 F/\partial y^2 + (eE(y)/W)\,\partial F/\partial y = [3c/(s_0 - s)]\,\partial F/\partial s. \tag{21}$$

The complex Fourier transform reduces the partial differential equation (21) to the subsidiary equation (22):

$$-p^2 F - (\rho y e/\varepsilon W)(p\,\partial F/\partial p + F) = [3c/(s_0 - s)]\,\partial F/\partial s. \tag{22}$$

The solution of equation (22) is given by the following expression of $F(p, s)$, the functions $F(0, s)$ and $a(s)$ are determined by calculating equation (22) for $F(p, s)$:

$$F(p, s) = F(0, s)\exp(-\tfrac{1}{2}a^2(s)p^2). \tag{23}$$

It is also interesting to note that in the case of charged targets, the electron deposition is characterized by a Gaussian distribution. The treatment of this problem is exactly similar to the determination of the solution of the electron diffusion equation in conducting materials and we can repeat the same procedure for calculating $\langle x\rangle$ and $\langle x^2\rangle$. We suppose a hypothetical electron having an x-coordinate given by the mean value of the electron distribution, then the loss of energy of this particle from the electric field is given by

$$W_1 = (\rho\langle x\rangle/\varepsilon)(\langle x\rangle/2 - \varepsilon_0 s_0/(\varepsilon + \varepsilon_0)). \tag{24}$$

Finally, we find that the correct solution for the semi-infinite charged medium is also given by the general expression (19), and we remark that this approach to the treatment of transport is well suited for such boundary conditions. Figure 2 illustrates the changes of electron distribution as charge builds up in a glass target. The chosen insulating specimen belongs to the ternary SiO_2–Cs_2O–Rb_2O system. Mixed alkali silica glasses are known to be very good insulators having superior combinations of properties than could be obtained with a binary alkali silica glass. The potential of the front surface of the target is allowed to float under the influence of the primary beam. There is evidence that a breakdown process depending on the target composition or gas adsorption on the target surface can be induced by the charge accumulation. By approximating the charged area to addition of thin charged discs with an axial symmetry, we calculate the surface potential as a function of the charging dose [32]. An electron dose of 1.2×10^{-8} C cm^{-2} is required to induce the surface potential of 4.5 keV in the case of curve A of figure 2. In the other cases of this figure the dose of the primary particles varies linearly with the surface potential. From figure 2 it is clear that the charge distribution varies asymmetrically as a consequence of the negative image charge, which has an important effect upon the field distribution. We can verify that the charging effect may be described by a reduction of the beam penetration of the primary electron. A comparison with the results of Kotera and Suga [33] describing the same situation shows general good agreement between this model and the simulation of electron trajectories in a charged specimen with an ungrounded surface. The irradiated volume decreases as the surface potential increases during the charging process. The experimentally observed charge build-up is more rapid than the calculated rate of charging during the same recording time. In fact the dynamics of the charging process is more adequately described by

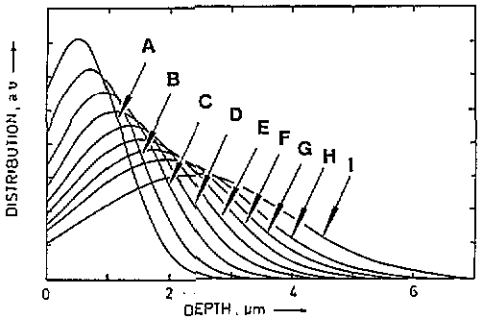

Figure 2. Depth distributions of electrons in a glass target plotted for several values of the surface potential: A, $V_0 = 4.5$ kV; B, $V_0 = 4$ kV; C, $V_0 = 3.5$ kV; D, $V_0 = 3$ kV; E, $V_0 = 2.5$ kV; F, $V_0 = 2$ kV; G, $V_0 = 1.5$ kV; H, $V_0 = 1$ kV; I, $V_0 = 0$ kV. The target surface is not at ground potential. The specimen belonging to the SiO_2–Cs_2O–Rb_2O system is irradiated by an electron beam energy of 30 kV.

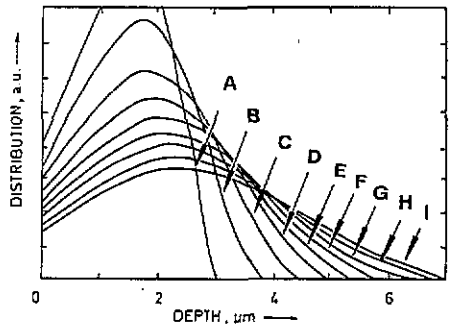

Figure 3. Electron distributions in a glass target covered by a conducting film in the presence of an electric field. The curves A, B, C, D, E, F, G, H and I are plotted for doses of incident electrons 125 times higher than in the irradiation used for plotting the corresponding distributions represented in figure 2. The irradiated specimen is a type of glass belonging to the SiO_2–Cs_2O–Rb_2O system and the energy of the incident particles is 30 keV.

introducing the notion of a time-dependent conductivity that rises from zero to a steady state conductivity and the time constant of the exponential variation of the conductivity is generally shorter than the recording time.

The transport equation has been extended in the above paragraph to the study of multiple scattering of electrons in charged targets. It shows that the electron distribution is the result of two defocusing processes originating, on the one hand, from the electric field and, on the other hand, from elastic collisions. Both actions tend to spread the lateral distribution into a Gaussian one and to decrease the range and the diffusion depth in the presence of a charging field.

6. Electron distribution in a grounded charged target

We now study the case where the insulating target is covered by a very thin grounded conducting film; this situation is consistent with the experimental operating conditions of practical interest for electron probe microanalysis. The conducting surface on top of the insulator modifies the electric field in the irradiated volume. The potential at the target surface is then zero and a large decelerating electric field appears in the region close to the bombarded surface.

The corresponding mathematical problem can be treated by considering that the dielectric constant of the medium in front of the sample is infinite. Then the electric field near the target surface is given by

$$E(x) = \rho\varepsilon^{-1}(x - s_0) = -2V_m z/s_0^2 \tag{25}$$

where $z = x - s_0$ and $V_m = -\rho s_0^2/2\varepsilon$ is the minimal potential in the target. This expression clearly shows that if a high electrical field is directed against the electron propagation near the surface then the electrons of the primary beam are pushed back

towards the surface of the grounded sample. The variation of the electron penetration is mainly the result of the potential distribution inside the region in front of the sample. The value of the electric field is increasing from a negative value at the surface and becomes negligible at a depth equal to the electron range in the target. We obtain the electron distribution by the solution of the transport equation in making the same diffusion approximation as before (21):

$$\partial^2 F/\partial z^2 + (eE(z)/W)\,\partial F/\partial z = [3c/(s_0 - s)]\,\partial F/\partial s. \tag{26}$$

One can conclude that the general expression (19) giving the electron distribution in a semi-infinite target is also valid in the case of a grounded target.

The above calculation made for a grounded target shows a maximum negative potential much smaller than for an ungrounded target for the same dose of charges deposited. Figure 3 shows the charge distributions in a grounded glass target belonging to the SiO_2–Cs_2O–Rb_2O system. The calculated distributions explain fairly well the fact that all the curves that are calculated with the charging conditions represented on figure 2 are coincident if the target is grounded. An electron dose of 1.5×10^{-6} C cm^{-2} is required to induce the electron distribution represented by curve A of figure 3. Thus the potential distribution arising from the charges deposited in the target is a small perturbation if the specimen surface is coated by a thin metal layer at ground potential. In addition, the conductivity of an insulator is increased under electron irradiation and thus the excess migrating charges can be more easily evacuated. This remark contributes to the justification of the deposition of conducting films on the insulating specimens commonly used in electron probe microanalysis. Because of the rapid charge build-up, in experimental situations, the number of charges trapped over the range 3–6 μm of curve A has a negligible influence on the steady state of the electron distribution.

7. Conclusions

The theoretical study of charge behaviour produced by electron beam irradiation is a fruitful field of research to elucidate some problems encountered in various spectrometries such as energy shift spectra, electromigration of ions, distortion of images, and deflection of primary beams. We have evaluated the effects of beam-induced charging of a target material on the distribution of the steady state electron implantation. The Boltzman transport equation is able to predict the effects of the charging of the sample on the electron propagation in the target. Decreases of the mean penetration depth and of the range of the electron, arising from a decelerating field, are substantial in the case of irradiation of bulk samples in the presence of the electric field created by the distribution of trapped charges. This result is in agreement with the experimental description of the motion of ionic species observed by many authors in Auger electron spectroscopy. Calculations based on this model can satisfactorily clarify many aspects of electron-induced charging related to the electron penetration and to the variance of the electron distribution. It can be concluded that the knowledge of these parameters is significant for a better interpretation of the electron probe microanalysis or Auger electron analysis results and permits the experimentalist to change the operating parameters such as accelerating voltage, probe inclination or size, in order to optimize the analysis conditions.

References

[1] Battaglin G, Della Mea G, de Marchi G and Mazzoldi P 1982 *Rad. Eff.* **64** 99
[2] Knotex M L and Feibelman P J 1978 *Phys. Rev. Lett.* **40** 964
[3] Ohuchi F and Holloway P 1982 *J. Vac. Technol. Lett.* **20** 863
[4] Miotello A and Mazzoldi P 1985 *Phys. Rev.* **54** 1675
[5] Jacob J 1973 *Phys. Rev.* **8** 226
[6] Marshak R E 1947 *Phys. Rev.* **71** 443
[7] Williams E J 1940 *Phys. Rev.* **58** 292
[8] Chandrasekhar S 1950 *Radiative Transfer* (Oxford: Clarendon) p 50
[9] Goudsmit S and Saunderson J L 1940 *Phys. Rev.* **54** 24
[10] Weymouth J W 1951 *Phys. Rev.* **84** 766
[11] Aronson R 1970 *Phys. Rev.* **11** 931
[12] Wang M C and Guth E 1951 *Phys. Rev.* **84** 1092
[13] Hemenger R P 1974 *Phys. Rev.* **64** 503
[14] Wick G C 1949 *Phys. Rev.* **75** 738
[15] Biagy S F 1988 *Nucl. Instrum. Methods* A **273** 533
[16] Archard G D and Mulvey T 1977 *Br. J. Appl. Phys.* **10** 7
[17] Love G, Cox M G C and Scott V D 1977 *J. Phys. D: Appl. Phys.* **10** 7
[18] Brown D B and Ogilvie R E 1966 *J. Appl. Phys.* **37** 4429
[19] Vigouroux J P, Durand J P, Le Moel A, Le Gressus C and Griscom D L 1985 *J. Appl. Phys.* **57** 5139
[20] Anderson D B 1973 *J. Appl. Phys.* **47** 1513
[21] Smith R C 1972 *Appl. Phys. Lett.* **21** 352
[22] Fatthers D J and Rez P 1984 *Proc. Ist Pfefferkorn Conf., (Monterey, USA)* ed A M F O'Hare (Chicago, IL: SEM) p 193
[23] Mott N and Massey H 1949 *Theory of Atomic Collisions* (Oxford: Oxford University Press) p 135
[24] Lewis H W 1950 *Phys. Rev.* **78** 526
[25] Bethe H A 1953 *Phys. Rev.* **89** 1256
[26] Bethe H A, Rose M E and Smith L P 1938 *Proc. Am. Phil. Soc.* **78** 573
[27] Kanaya K, Ono S and Ishigaki F 1978 *J. Phys. D: Appl. Phys.* **11** 2425
[28] Meister H 1958 *Z. Naturf.* **13** 809
[29] Carslaw H S and Jaeger J C 1959 *Conductivity of Heat in Solids* (Oxford: Clarendon) p 353
[30] Bethe H A and Jacob J H 1977 *Phys. Rev.* **16** 1952
[31] Cazaux J 1986 *J. Appl. Phys.* **59** 1418
[32] Kotera K and Suga H-1986 *J. Appl. Phys.* **63** 261
[33] Landron C 1988 *J. Appl. Phys.* **63** 586